Homework 2

P2.1.4 The voltage shown in Figure P2.1.4 is applied across a 5 Ω resistor. Determine: (a) the resistor current.; (b) $p(t)$, $0 \le t \le 1\text{min}$;

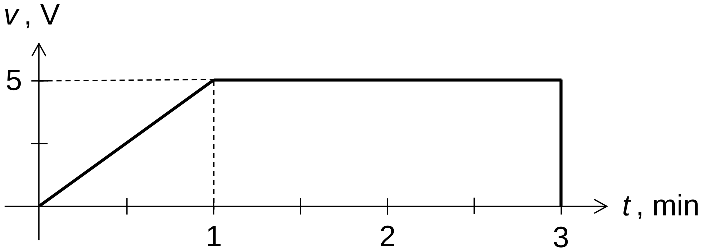


Figure P2.1.4

(c) the energy dissipated in the resistor at t = 3 min.

Solution: $v\text{ (V)} = \begin{cases} t/12 & 0 \le t \le 60\text{ s} \\ 5 & 60 \le t < 180\text{ s} \\ 0 & t > 180\text{ s} \end{cases}$

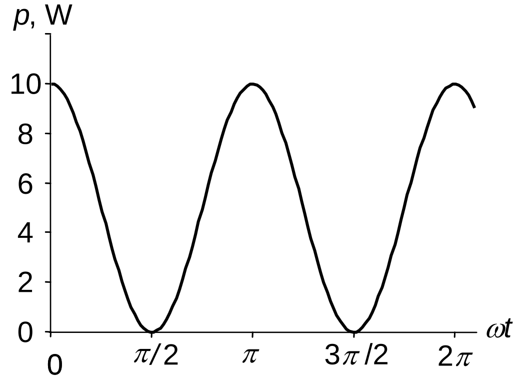
(a) $i = v/R$; $i = t/60$ A, $0 \le t \le 60$ s; i = 1 A, $60 \le t < 180$ s; i = 0, $t > 80$ s.

(b) $0 \le t \le 60\text{ s}$; $p(t) = \dfrac{v^2}{R} = \dfrac{1}{5}\left(\dfrac{t}{12}\right)^2 = \dfrac{t^2}{720}$ W, where t is in s.

(c) $w = \int_0^{180} p dt = \int_0^{60} \dfrac{t^2}{720} dt + \int_{60}^{180} \dfrac{25}{5} dt = 700$ J.

P2.1.5 A voltage $v(t) = 10\cos 100\pi t$ V is applied across a 10 Ω resistor. (a) Sketch $p(t)$. (b) Determine the average power dissipated in the resistor and the energy dissipated during half a cycle of $v(t)$.

Solution: $v(t) = 10\cos 100\pi t = 10\cos\omega t$, where $\omega = 100\pi$ rad/s, and so that the supply frequency is $\frac{100\pi}{2\pi} = 50$ Hz, and the supply period is $\frac{1}{50} \equiv 20$ ms.



$$f = \frac{w}{2\pi}\ where\ f\ is\ in\ Hz\ and\ w\ is\ in \frac{rad}{2\pi}$$

(a) $p = \frac{v^2}{R} = \frac{(10\cos 100\pi t)^2}{10}$ $\cos^2\alpha = (1 + cos2\alpha + \frac{1}{2} = 10\cos^2 100\pi t$ = 5(1 + cos2ωt) W, as shown.

(b $= \frac{5}{2\pi}\int_0^{2\pi}(1 + \cos 2\omega t)d(\omega t) = \frac{10}{2\pi}\times\frac{2\pi}{2} = 5$ W

$$w = \int_0^{0.01} pdt = \int_0^{0.01} 10\cos^2 100\pi t dt = 5\int_0^{0.01}(1 + \cos 200\pi t)dt$$

$= 5\left[t + \frac{\sin 200\pi t}{200\pi}\right]_0^{0.01} = 0.05$ J. Since the average power dissipated is 5 W, the energy dissipated during one half cycle is 5(W)×0.01(s) = 0.05 J. Note that the average power, i.e., average energy per unit time, is independent of the time scale, but the energy is the integral of instantaneous power with respect to time.

P2.2.6 Determine in Figure P2.2.6 the voltage across each current source, the current through each voltage source, and the power delivered or absorbed by each source.

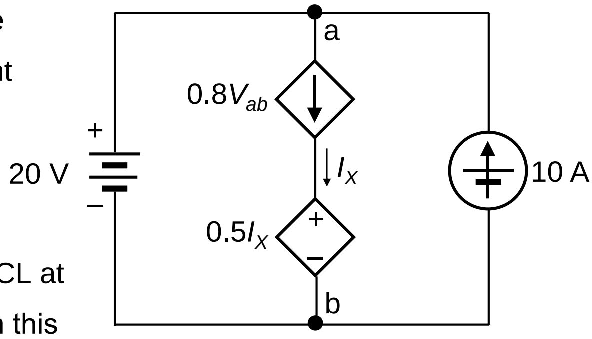


Figure P2.2.6

Solution: V_{ab} = 20 V, I_X = 0.8V_{ab} = 16 A. From KCL at node 'a', net current flowing away from this node is 16 – 10 = 6 A. Hence, 6A must flow into node 'a' from the 20 V source.

Voltage drop across the CCVS = 0.5I_x = 8 V; let the voltage drop across the VCCS be V_X. From KVL the voltage drop V_{ab} is the same whether going through the 20 V source or the two dependent sources. Hence 20 = V_X + 8, which gives V_X = 12 V.

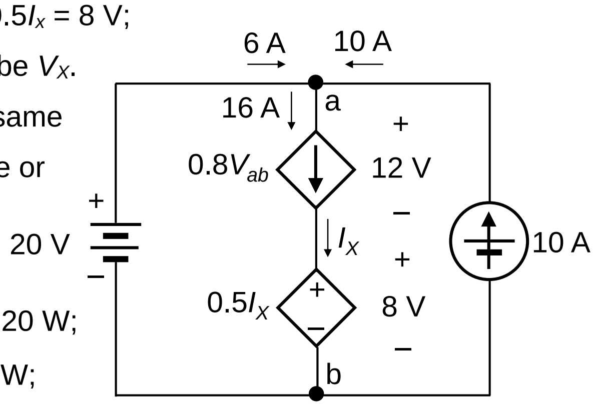


Figure P2.2.6-1

The 20 V source delivers 20×6 = 120 W; the 10 A source delivers 20×10 = 200 W; VCCS absorbs 12×16 = 192 W; and CCVS absorbs 8×16 = 128 W. Total power delivered = 320 W = total power absorbed.

P2.2.7 Determine the total power delivered or absorbed by each source in Figure P2.2.7, assuming the voltage sources are 1 V each, I_1 = 2 A, I_2 = 1 A, and I_3 = 1 A.

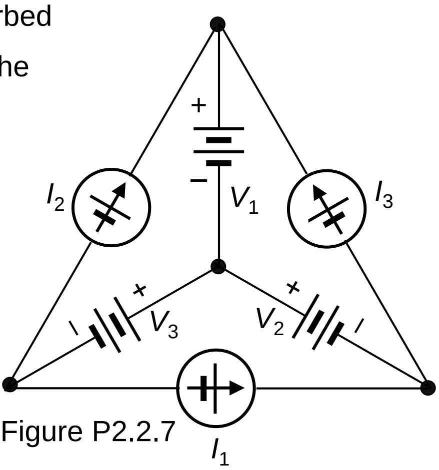


Figure P2.2.7

Solution: The current leaving node 'a' through V_1 is $I_2 + I_3 = 1 + 1 = 2$ A. The source V_1 absorbs $1\times2 = 2$ W.

The current leaving node 'b' through V_2 is $I_1 - I_3 = 2 - 1 = 1$ A. The source V_2 delivers $1\times1 = 1$ W.

The current entering node 'c' through V_3 is $I_1 + I_2 = 3$ A. V_3 absorbs $1\times3 = 3$ W.

From KVL, $V_{ab} = 2$ V, $V_{ac} = 2$ V, and $V_{bc} = 0$. The source I_1 neither absorbs nor delivers power; the source I_2 delivers 2 W; the source I_3 delivers 2 W.

Total power delivered = 5 W = total power absorbed.

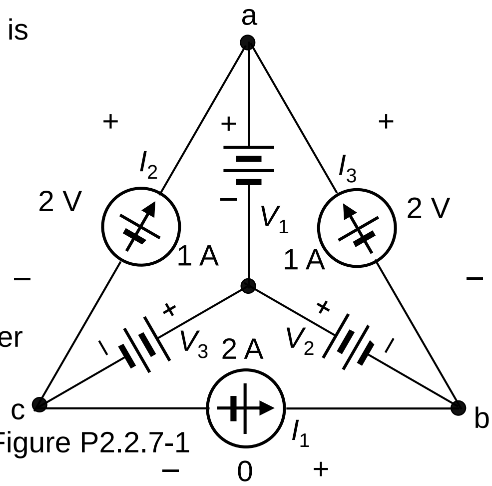


Figure P2.2.7-1

P2.2.8 Determine in Figure P2.2.8 the voltage across each current source, the current through each voltage source, and the power delivered or absorbed by each source.

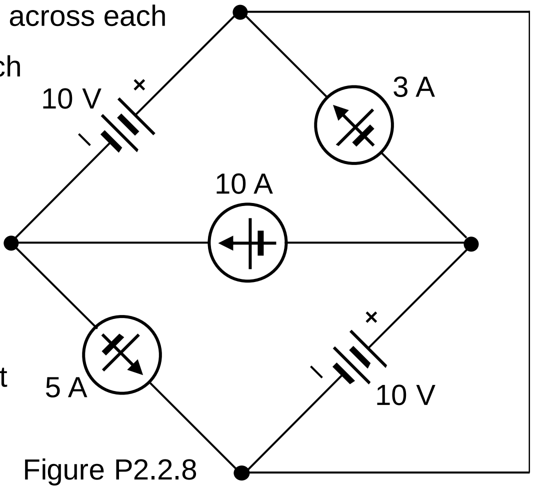


Figure P2.2.8

Solution: From KCL at node ‘d’: 10 = 5 + I_Δ, so I_Δ = 5 A. From KCL at node ‘b’: I_Y = 10 + 3 = 13 A. From KCL at node ‘a’: 3 + 5 = 8 A = I_X. As a check, KCL at node ‘c’ is: 13 = 8 + 5 A.

From KVL around the outer loop: $-V_{cd}$ + 10 = 0, so that V_{cd} = 10 V. From KVL around the mesh on the RHS: 10 – V_{ba} = 0, so that V_{ba} = 10 V. From KVL around the mesh ‘abda’: $-V_{ba}$ – 10 + V_{bd} = 0, so that V_{bd} = 20 V. As a check, KVL around the mesh ‘bcdb’ gives: -10 – V_{cd} + V_{bd} = -10 – 10 + 20 = 0. Upper 10 V source delivers 50 W; lower 10 V source delivers 130 W; 5 A source delivers 50 W; 10 A source absorbs 200 W; 3 A source absorbs 30 W. Total power delivered = 230 W = total power absorbed.

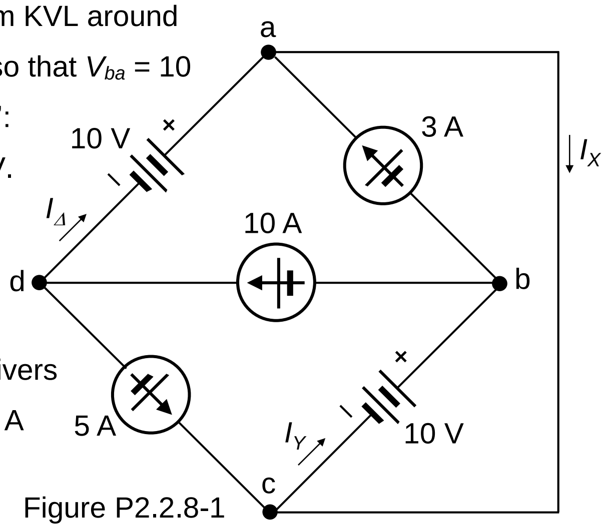


Figure P2.2.8-1

P2.3.23 Determine the power delivered or absorbed by the dependent source in Figure P2.3.23.

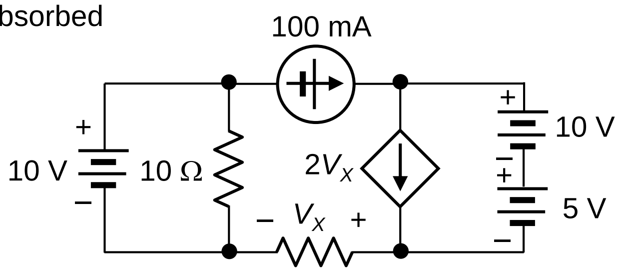


Figure P2.3.23

Solution: **Initialize.** The circuit is marked with given values. To determine the power delivered or absorbed by the dependent source, the voltage V_X should be determined.

Simplify. The circuit is in a simple enough form, although the two batteries can be combined in one 15 V battery.

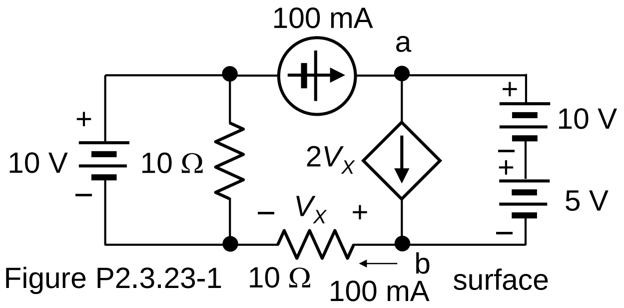


Figure P2.3.23-1

Deduce. From KCL around a enclosing the dependent source and the two batteries, the current through the lower 10 Ω resistor is 100 mA in the direction shown. From Ohm's law, $V_X = 10\times0.1 = 1$ V. The source current of the dependent source is 2 A. Since the voltage across the source is 15 V, and the 2 A is in the direction of a 15 V drop across the source, the source absorbs $15\times2 = 30$ W.

P2.3.31 Determine R, V_X, and V_Y in Figure P2.3.31, given that the current in the top connection is zero.

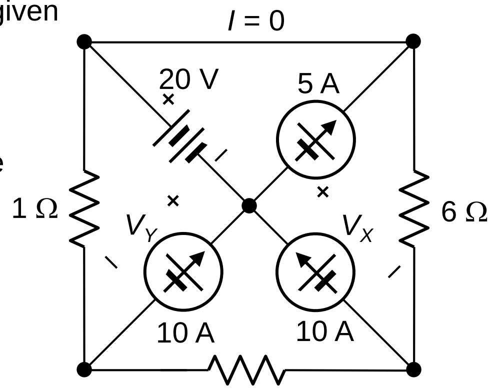


Figure P2.3.31

Solution: **Initialize.** The circuit is marked with given values. The nodes are labeled. To determine R, the voltage across it and the current through it should be determined. V_X and V_Y are determined from KVL around a mesh.

Simplify. The circuit is in a simple enough form.

Deduce. From KCL at node ‘b’, I_{be} = 5 A; from KCL at node ‘e’, I_{de} = 5 A; from KCL at node ‘c’, I_{ca} = 15 A; from KCL at node ‘a’, I_{ad} = 15 A; as a check, KCL is satisfied at node ‘d’. From Ohm’s law, V_{be} = 30 V, V_{ad} = 15 V, and V_{de} = 5R. From KVL in the outer loop, 5R + 15 – 30 = 0, which gives R = 3 Ω. From KVL around the mesh ‘abca’, -V_1 + 20 = 0, or V_1 = 20 V; from KVL around the mesh ‘cbec’, V_1 – 30 + V_x = 0, or V_x = 10 V. From KVL around the mesh ‘acda’, -20 – V_y + 15 = 0, or V_y = -5 V.

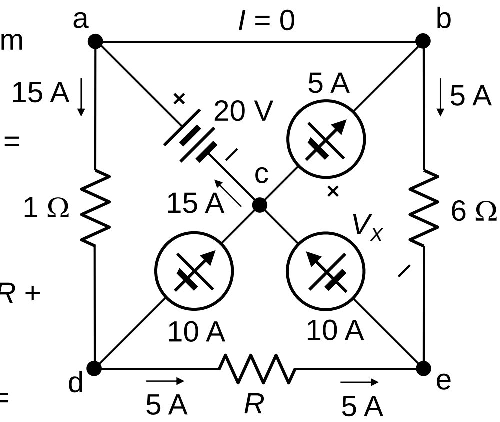


Figure P2.3.31-1

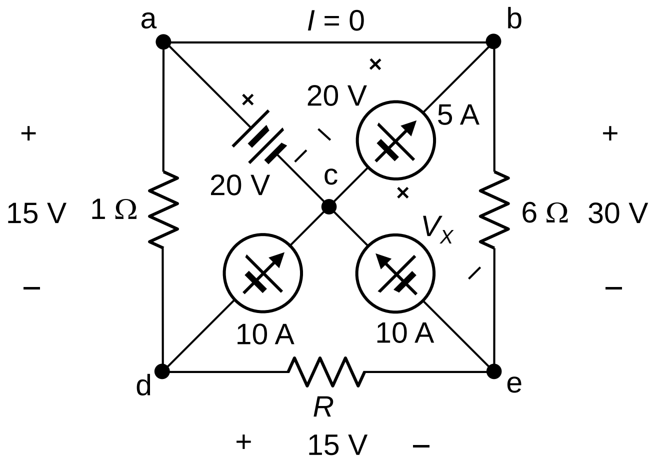


Figure P2.3.31-2

P2.3.32 Determine I_O in Figure P2.3.32.

Solution: **Initialize.** The circuit is marked with given values. The nodes are labeled.

Simplify. The circuit is in a simple enough form.

Deduce. $V_{ab} = 2I_O$, so that the current in the 2 Ω resistor is I_O. From KCL at node 'a', $I_{da} = 3I_O$. From KCL at node 'b', $I_{bc} = 0$. From KVL around the mesh 'abcd', $-2I_O + 0 + 5 - 3I_O = 0$, which gives $I_O = 1$ A.

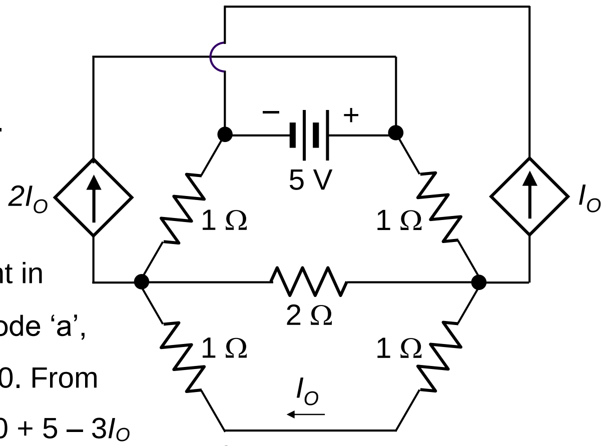


Figure P2.3.32

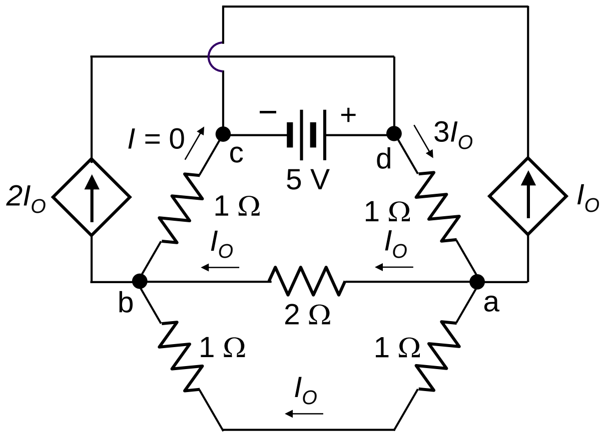


Figure P2.3.32-1